

Symmetrization, quantum images and measurement

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Abstract. We argue that symmetrization of an incoming microstate with similar states in a sea of microstates contained in a macroscopic detector can produce an effective image, which does not contradict the no-cloning theorem, and such a combinatorial set can then be used with first passage random walk interactions suggested in an earlier work to give the right quantum mechanical weight for measured eigenvalues.

1. Introduction

In a previous work [1] we have presented a picture of the transition of a superposed quantum microstate to an eigenstate of a measured operator through interactions with a measuring device, which are random in the sense of the stochasticity introduced by the large number of degrees of freedom of the macrosystem, and not due to any intrinsic quantum indeterminism. Random walks [2] have long been a favorite sports enjoyed by many quantum physicists in search of a rationale for quantum indeterminism [3]. Different stochastic models for transitions to collapsed states on measurement have been presented by many authors [4, 5, 6, 7, 8]. However, in our work we made the novel departure of using first passage walks [9] which lead to a dimensional reduction of the path in simplicial complexes to simplexes of lower dimensions by turn, a possible feature also noted very recently by Omnès [10]. In the work cited we appealed to heuristic arguments in analogy with electrodynamic images. In the present work we try to justify the emergence of image-like subsystems in a macrosystem from quantum symmetry principles.

2. Symmetrization and Interactions

Interactions between systems may be due to Hamiltonians connecting operators that explicitly connect components of different systems, or they may be due to symmetrization or anti-symmetrization of the states of the systems involved. For fermions, exchange interaction yields the exclusion principle, which may have more dominant effects than a weak potential in a many-particle system. For bosonic systems condensation at low temperatures indicate the creation of macro-sized quantum states. Unlike the unitary time-dependent operators representing the explicit interactions between systems through the Hamiltonian, (anti-)symmetrization has no explicit time involvement, and a system includes the (anti-)symmetrization of the component subsystems *ab initio*, which continues until the states change and lose their indistinguishability. Alternatively, (anti-)symmetrization comes into action as soon as an intermediate or final state is produced involving identical particles, even when the initial system might not have had any. The process therefore is apparently a discrete phenomenon, going together with the abrupt action of the creation or annihilation of particles in field theory.

In terms of first quantized quantum mechanics, we understand the permutative (anti-)symmetry properties of identical microstates (particles) in terms of the separation of the co-ordinates). Two identical microsystems labeled 1 and 2 in a particular states *a* and *b* has the combined (anti)symmetric wave function

$$\psi(1, 2)_{ab} = \frac{1}{\sqrt{2}}[\psi(1)_a\psi(2)_b \pm \psi(2)_a\psi(1)_b] \quad (1)$$

In practice the labels 1 and 2 for the two particles usually refer to the concentration of the two particles in two different regions of space, for example, near two attractive

potential centers. So, the labels 1 and 2 are actually also interpretable as parameters for two different states, and [11] it is possible to combine the two sets of labels into a single set, say α and β and demand that

$$\psi(\alpha, \beta) = \pm \psi(\alpha', \beta') \quad (2)$$

where the sign for fermionic systems depends on the number of interchanges needed to obtain the parameter sets α' and β' from the unprimed sets and for the bosons it is of course always positive.

Even with *ab initio* symmetry built-in, it is well-known that a state can dynamically evolve from a nearly factorized separable product state to a fully symmetric entangled state as the overlap becomes high from nearly zero when the two subsystems (particles) were well-separated initially. If we know that the incoming particles labeled 1 and 2 were in states a and b at large separations then,

$$\begin{aligned} \psi(x_1, x_2, a, b) &= \frac{1}{\sqrt{2}} [\psi(x_1, a)\psi(x_2, b) \pm \psi(x_2, a)\psi(x_1, b)] \\ &\sim \frac{1}{\sqrt{2}} \psi(x_1, a)\psi(x_2, b) \end{aligned} \quad (3)$$

for $|x_1 - x_2|$ large, as the second term is small .

If the states a and b are identical, then it is well-known that this exchange interaction for bosons gives an effective attractive interaction for small $|x_1 - x_2|$, as we get simply $\sqrt{2}$ times a single wave function, whereas for fermions it becomes highly repulsive as the antisymmetry produces the exclusion principle.

3. State of the Detector

We shall consider a detector as macrosystem which consists of a large number of microsystems identical with the microsystem to be detected, but in all possible different states, including the incoming state to be detected, so that initially it appears like a neutral unbiased system with respect to the state of the incoming microsystem. This picture is comparable to that of a sea of quarks of all flavors and colors in a quark bag, or even the similar content of a neutral vacuum when considering vacuum polarization contributions. To maintain the quantum number of the vacuum, i.e. to give a singlet with respect to all possible symmetry/classification groups, all these states occur paired with conjugate anti-states (group theoretically inverse elements):

$$\Psi_D = \sum_a \psi_{Da} \bar{\psi}_{Da} \quad (4)$$

where the label D indicates states with positional peaks inside the detector. The expression above is the simplest spectral decomposition for our purpose. In general there will also be simultaneous multiple state/anti-state pairs, which will introduce new numerical factors from combinatorics, but will not change the relative strengths of interactions between the incoming microstate (S) and the pairing anti-states of the detector (D), which is the crucial part of our measurement picture.

4. System-Detector Symmetry

For a bosonic microsystem system being detected, if it is in the state ψ_{Si} , symmetrization with the detector states gives

$$\psi_{SD} = \frac{1}{\sqrt{2N}} \sum_{j \neq i} (\psi_{Si} \psi_{Dj} + \psi_{Sj} \psi_{Di}) \bar{\psi}_{Dj} + \frac{1}{\sqrt{N}} (\psi_{Si} \psi_{Di} \bar{\psi}_{Di}) \quad (5)$$

when there are N states uniformly distributed in the detector, including the state i . Normalization is ensured by the orthogonality of the states, when the coefficients are as chosen.

However, if the microsystem was well-separated from the detector and symmetrization was not invoked, the product state in a product space would be, with the macroscopic detector still containing a superposition of all possible states:

$$\Psi_{SD0} = \frac{1}{\sqrt{N}} \sum_{allj} (\psi_{Si} \psi_{Dj} \bar{\psi}_{Dj}) \quad (6)$$

Since the functions ψ_{Si} and ψ_{Di} for an identical microsystem in the same state may both actually represent the observed incoming microsystem in Eq. 6, we can rewrite Eq. 5 as

$$\Psi_{SD} = \frac{1}{\sqrt{2}} (\Psi_{SD0} + \Psi_{DS0}) + \frac{1}{\sqrt{N}} (1 - \sqrt{2}) \psi_{Si} \psi_{Di} \bar{\psi}_{Di} \quad (7)$$

Here both Ψ_{SD0} and Ψ_{DS0} represent an incoming particle in the state ψ_i and its noninteracting product with the detector. Hence, the extra term $\psi_{Si} \psi_{Di} \psi_{Di}$ represents the 'exchange interaction' resulting from the entanglement of the microstate with the detector.

For incoming fermionic systems the arguments are similar, but somewhat more complicated. In this case anti-symmetrization gives

$$\Psi_{[SD]} = \frac{1}{\sqrt{2(N-1)}} \sum_{j \neq i} (\psi_{Si} \psi_{Dj} - \psi_{Sj} \psi_{Di}) \bar{\psi}_{Dj} \quad (8)$$

Since the detector includes all other states but must exclude the state ψ_i due to anti-symmetrization (exclusion principle), we can actually consider the sums in Eq. 8 as involving hole-antihole pair states $\psi_{Di}^h \bar{\psi}_{Di}^h$ corresponding to ψ_i . So we have for the combined system of the incoming microsystem ψ_i and the detector:

$$\Psi_{[SD]} \sim (\psi_{Si} \psi_{Di}^h - \psi_{Di} \psi_{Si}^h) \bar{\psi}_{Di}^h \quad (9)$$

,

with the definitions:

$$\begin{aligned} \psi_{Di}^h \bar{\psi}_{Di}^h &= \sum_{j \neq i} \psi_{Dj} \bar{\psi}_{Dj} \\ \psi_{Si}^h \bar{\psi}_{Di}^h &= \sum_{j \neq i} \psi_{Sj} \bar{\psi}_{Dj} \end{aligned} \quad (10)$$

In the above analysis we have not considered the eigen-basis of the detector. As we have considered the symmetrization aspects only, the state ψ_i occurs as a natural preferred vector and for the other states $j \neq i$ we can consider any set orthogonal to ψ_i .

5. Quantum Images and the No-cloning Theorem

The exchange interaction term due to (anti-)symmetrization contains a product of the incoming microstate ψ_{Si} , a corresponding state ψ_{Di} in the detector, which is the same microstate for bosons, or a hole ψ_{Di}^h in the case of fermions, and also associated with such a pair is a conjugate state $\bar{\psi}_{Di}$ or $\bar{\psi}_{Di}^h$ for fermions. In the case of the bosonic systems we shall call the latter conjugate state an image of the original incoming state created by the symmetrization process. We do not consider the symmetric identical state ψ_{Di}^i as the image, because the identical state nominally in the detector is *indistinguishable* from the original incoming state and when there is an overlap of functions they may represent the same physical entity. In the case of fermionic systems the incoming state ψ_{Si} and the corresponding hole state ψ_{Di}^h or its conjugate $\bar{\psi}_{Di}^h$ are in general all nonidentical systems. Since the incoming state is definitely not ψ_{Si}^h , we can neglect the second term in Eq. 9. Hence the effect of the antisymmetrization effectively gives a simple product as for a bosonic system:

$$\Psi_{SD_{ferm}} = \psi_{Si} \psi_{Di}^h \bar{\psi}_{Di}^h \quad (11)$$

However, since the hole is more like a conjugate and the conjugate of the hole is more like the original incoming microsystem, we can expect that both ψ_{Si} and $\bar{\psi}_{Di}^h$ interact in a similar manner with ψ_{Di}^h .

There is no conflict with the no-cloning theorem [12] when (anti-)symmetrization produces such quantum images, which, as we have seen, are either extensions of the original functions, or are conjugate states. Though there is a one-to-one correspondence with the incoming state, the states in the detector simply extend the original state by (anti-)symmetry or produce a state which is conjugate to the original state, and is not producible by a unitary operator assumed in the no-cloning theorem. In other words, (anti-)symmetrization and the consequent exchange interactions are not producible by the linear unitary operators and the simple and elegant proof of the no-cloning theorem is inappropriate for quantum images of the kind described above.

6. Measurement and Eigenstates

Quantum images, formed by invoking symmetrization properties of the combined system, do not depend on the operator involved in the measurement process associated with the detector. The quantity measured is represented by a unitary operator in quantum mechanics, and, if the microsystem is an eigenstate, it remains in the same state even after measurement, but if it is a mixture of eigenstates of the operator, then it is taken as a postulate of quantum mechanics that the emerging state after measurement

is one of the eigenstates and the detector too carries off the information of the final state to which it collapses. We have shown recently [1] how a first passage random walk model reduces an arbitrary linear combination of eigenstates to one of the component eigenstates with a probability proportional to the square of the absolute magnitude of the coefficient of that component. In that work we appealed to an electrostatic analogy for the formation of the image in the detector which interacts with the incoming microsystem in steps, both changing simultaneously till an eigenstate is reached.

If the state ψ_{Si} is expressed in terms of the eigenstates in a simple two-state system

$$\psi_{Si} = a_i|\alpha\rangle_S + b_i|\beta\rangle_S \quad (12)$$

then we get

$$\psi_{Di} = a_i|\alpha\rangle_D + b_i|\beta\rangle_D \quad (13)$$

and

$$\bar{\psi}_{Di} = a_i^*|\bar{\alpha}\rangle_D + b_i^*|\bar{\beta}\rangle_D \quad (14)$$

and similarly for the hole states in the case of the fermionic systems.

This shows how the complex conjugate of the co-efficients occur in a natural way in the image, which is not possible by cloning with a unitary operator.

Here we also see that the conjugate can interact interchangeably with the incoming state or its indistinguishable extension in the detector and form virtual bound states

$$\begin{aligned} |SD\rangle_i &\sim |a_i|^2|\alpha\rangle_S|\bar{\alpha}\rangle_D + |b_i|^2|\beta\rangle_S|\bar{\beta}\rangle_D \\ |DD\rangle_i &\sim |a_i|^2|\alpha\rangle_D|\bar{\alpha}\rangle_D + |b_i|^2|\beta\rangle_D|\bar{\beta}\rangle_D \end{aligned} \quad (15)$$

We have shown [1] that first passage random walk in Hilbert space with the modulus squared of the coefficients of the eigenstate components of the microstate ($|a_\alpha|^2$, $|b_\beta|^2$, ...) as co-ordinates lead to unique final eigenstates $|\alpha\rangle$, $|\beta\rangle$, ..., with probabilities proportional to the corresponding initial co-ordinates ($|a_\alpha|^2$, $|b_\beta|^2$, ...), as expected. The constant of proportionality is a measure of the efficiency of the measuring device.

7. Conclusions

We have shown above that if the detector is a macroscopic system and is initially neutral with respect to the measured quantity, which we have expressed as the sum of microstates with all different states, then symmetrization with the measured system for bosonic systems or anti-symmetrization for fermionic systems breaks the neutrality in a unique way which may be regarded as the formation of a quantum image of the measured microsystem in the detector. These images are conjugates of the incoming microsystems, or hole-type states equivalent to conjugate states, and since the process is not a linear unitary operation, the no-cloning theorem does not pose a problem. That the interaction between the incoming state and these images can be modeled by first

passage random walks to give the probabilities for different eigenstates as final states of both the incoming state and the detector's microstate component has been shown in [1]. We shall later examine the question of measurement of entangled systems in spatially separated detectors.

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